→ Transport II

→ review structure of Landau Collision Integral

→ introduce "Rosenbluth Potentials"

→ discussion drag, diffn. as central to
- slowing down
- spreading

→ Beam Scattering $\begin{cases} \tau_s \\ \tau_D \end{cases}$

→ Resistivity

→ Runaways

→ Some Scalings

→ Dynamic Screening

→ General Structure of Transport Problem

– by either { Boltzmann + small deflection (class) → Landau
{ Fokker-Planck (Kulsrud)

⇒ arrive at: Landau Collision Integral

$$\frac{\partial \langle f \rangle}{\partial t} = -\frac{\partial}{\partial \underline{v}} \cdot \underline{J}(\underline{v})$$

dynamical friction — diffusion

$$\underline{J}(\underline{v}) = \sum_{\text{spec}} \int d^3v' \, B_{\alpha\beta} \left[\frac{\partial f(v')}{\partial v'_\beta} f(\underline{v}) - f(v') \frac{\partial f(\underline{v})}{\partial v_\beta} \right]$$

interaction; field particle dist; test particle distribution

$$B_{\alpha\beta} = \frac{2\pi (q q')^2}{m^2 |v - v'|} \ln \Lambda \left[\delta_{\alpha\beta} - \frac{v_\alpha v_\beta}{v_{rel}^2} \right]$$

(rel rel) — cutoff

Now can re-group in form: (test distribution evolution)

dynamical friction — diffusion

$$\left. \frac{\partial \langle f \rangle}{\partial t} \right|_c = + \sum_{\text{species }\alpha} \left\{ -\frac{\partial}{\partial v_j} \left(\frac{\partial h_\alpha}{\partial v_i} f_{Test} \right) + \frac{1}{2} \frac{\partial^2}{\partial v_i \partial v_j} \left(f_{Test} \frac{\partial^2 g_\alpha}{\partial v_i \partial v_j} \right) \right\}$$

$$* \left(\frac{4\pi n_\alpha q_T^2 q_\alpha^2}{m_T^2} \ln \Lambda \right)$$

here:

h_α, g_α } "Rosenbluth potentials" in terms:
$f(\underline{v}_\alpha)$
↳ distribution of field particles

where:
$$h_\alpha(\underline{v}) = \frac{m_T}{\mu_{\alpha'}} \int d^3v' \, \frac{f_\alpha(\underline{v}')}{|\underline{v}-\underline{v}'|}$$

$$g_\alpha(\underline{v}) = \int d^3v' \, f(\underline{v}') \, |\underline{v}-\underline{v}'|$$

loosely analogous to electrostatics (v' integrated)

Now can: – choose field distribution (i.e. background)
– set test problem (i.e. beam)

$\Rightarrow$ compute evolution – i.e. beam slow down

Will consider several examples...

N.B. Rosenbluth potentials reduce Landau problem/collision operator to "plug-and-chug".

i.e. (1) identify field particle distribution

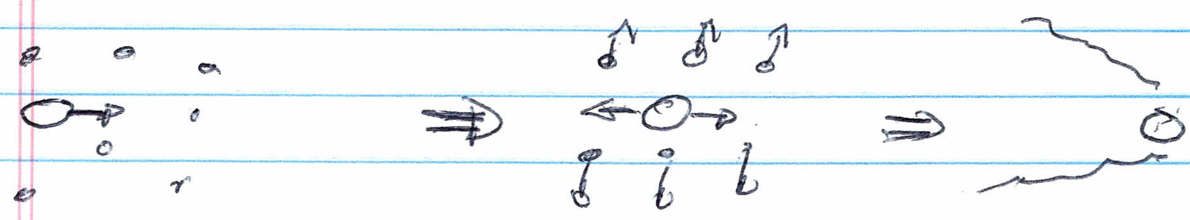
(2) compute h, g

(3) solve for $\partial_t \langle f \rangle_v$ and its moments, i.e.

$\partial_t \overline{V} / \overline{V} \rightarrow 1/\tau_s \Rightarrow$ slowing down time

$\partial_t \langle V^2 \rangle / \langle V^2 \rangle \rightarrow 1/\tau_D \Rightarrow$ diffusion time

→ what is dynamical friction?



particle forms a wake:

- series of glancing collisions, scattering field particles laterally, but due balance, no net deflection of test particle

- some ⓒ direct scattering of test particle ⇒ slow down.

Applications of Collision Theory

Recall, have Landau – Fokker-Planck – Rosenbluth Equation:

$$\left.\frac{\partial \langle f \rangle}{\partial t}\right)_c = \sum_{\substack{\text{spcs} \\ \alpha}} -\frac{\partial}{\partial v_i}\left[\left\{ \frac{\partial h_\alpha}{\partial v_i} f_{test} - \frac{1}{2}\frac{\partial}{\partial v_j}\left(\frac{\partial^2 g_\alpha}{\partial v_i \partial v_j} f_{Test}\right)\right\} * \left(\frac{4\pi n_\alpha}{m} q_T q_\alpha \ln \Lambda\right)\right]$$

$\alpha \Rightarrow$ field particle species (i.e)

$i \rightarrow$ test particle species

$\left.\begin{matrix} h_\alpha \\ g_\alpha \end{matrix}\right\}$ Rosenbluth potentials $\equiv$ functionals of field particle distribution $f(v)$

$$h_\alpha(\underline{v}) = \frac{m_T}{\mu_\alpha}\int d^3v' \frac{f_\alpha(\underline{v}')}{|\underline{v}-\underline{v}'|}$$

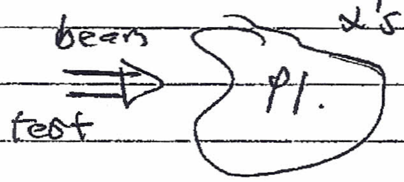
$$g_\alpha(\underline{v}) = \int d^3v' f(\underline{v}') |\underline{v}-\underline{v}'|$$

Now, consider applications:

(a) → deceleration of beam ⇒ dynamical friction
(b) → spreading of a beam ⇒ diffusion
(c) → Conductivity (Resistivity)
(d) → Runaway (the exception)

(a) Deceleration of Beam

Consider



$f_T(\underline{v}, t=0) = \delta(\underline{v}-\underline{v}_0)$ → test particles

Now, for deceleration, need: $\left.\frac{\partial}{\partial t}\underline{V}\right|_{macro}$

So taking $\int d^3v\, \underline{v}$ of FP ⇒

$$\boxed{\frac{\partial \underline{V}}{\partial t} = \sum_\alpha \gamma_\alpha \int d\underline{v}\, \bar{n}_\alpha \left(f_T \frac{\partial h_\alpha}{\partial \underline{v}} \right)}$$

⇒ sum of: b-e drag + b-i drag → set by dynamical friction

$\gamma_\alpha = 4\pi Z_T^2 q_\alpha^2 \ln\Lambda / m_T^2$

w $h_\alpha(\underline{v}) = \frac{m_T}{\mu_{T\alpha}} \int d^3v' \frac{f_\alpha(\underline{v}')}{|\underline{v}-\underline{v}'|}$

NB diffn contribution integrates out.

Test $\to \delta(\underline{v}-\underline{v}_0)$

Now:

$$\int d\underline{x}\, e^{-x^2} / |\underline{x}-\underline{y}| = \phi(y)\, \pi^{3/2} / y$$

$$\phi(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2}\, dx$$

useful identity (Field Maxwellian)

$\Rightarrow$ plugging:

$$\frac{\partial V}{\partial t} = \left(\frac{4\pi n_0 e^2 q_T^2 \ln\Lambda}{m_T^2}\right) \frac{\partial}{\partial V} \left[\underbrace{\frac{1}{V} \phi\left(V/v_{Te}\right)\left(1+\frac{m_T}{m_e}\right)}_{b-e} \underset{\sim 1/V^2}{} + \underbrace{\frac{Z}{V} \phi\left(U/v_{Ti}\right)\left(1+\frac{m_T}{m_i}\right)}_{b-i} \right]$$

$U \to$ beam

can define "slowing down time" by:

$$\boxed{\tau_S = -V\left(\frac{\partial V}{\partial t}\right)^{-1}}$$

$V \to$ beam speed

so

$\rightarrow$ if $V > v_{Te} > v_{Ti}$

$\rightarrow$ b-e scattering dominates slowing down

$\Rightarrow$ integrals

so

$$\tilde{\tau}_s \approx M_T V^3 / 4\pi n_e e^2 Z_T^2 (2+M_T/m_e)\ln\Lambda$$

- slowing & scattering longer for heavier
- insensitive temp
- $\tilde{\tau}_s \sim V^3$

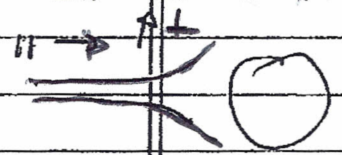
→ if $\quad v_{Te} > V > v_{Ti}$

$$\tilde{\tau}_s = M_T^2 / 4\pi n_0 e^2 Z_T^2 \left[\frac{2}{u^3}\left(1+\frac{m_T}{m_e}\right) + \frac{4}{\sqrt{3\pi}}\left(1+\frac{M_T}{m_e}\right)\frac{1}{v_{Te}^3} \right] \ln\Lambda$$

Temp dependence

• $V < v_{Ti} < v_{Te} \Rightarrow$ HW.

(b) Spreading of Beam — set by diffusion



$\partial_t V_\parallel^2$

$\partial_t V_\perp^2$

$$f_T = \delta(\underline{V}-\underline{v}) = \delta(\underline{V}_\parallel-\underline{v})$$

⇒ so, plugging into $\int d\underline{v}\ \underbrace{v^2}_{v_\parallel^2,\ v_\perp^2} \frac{\partial}{\partial t} f)_c$

gives:

(macro)

⇒ plugging in:

// diffn

$$\partial_t V_\parallel^2 = \frac{4\pi n_0 e^2 Z_T^2 \ln\Lambda}{m_T^2}\left[\frac{\partial}{\partial V_\parallel^2}\int d\underline{v}'\,(f_e + Zf_i)\,|\underline{V}-\underline{v}'| \right.$$

drag, g

$$\left. + 2V_\parallel \frac{\partial}{\partial V_\parallel}\sum_\alpha \left(1+\frac{m_T}{m_\alpha}\right)\int d\underline{v}'\,\frac{Z_\alpha f_\alpha(\underline{v}')}{|\underline{V}-\underline{v}'|}\right]$$

h

and

$$\partial_t V_\perp^2 = \frac{4\pi n_e e^2 Z_T^2 \ln\Lambda}{m_T^2}\left(\frac{\partial^2}{\partial V_{\perp_1}^2} + \frac{\partial^2}{\partial V_{\perp_2}^2}\right) *$$

perp diffn

$$\int d\underline{v}'\,(f_e + Zf_i)\,|\underline{V}-\underline{v}'|$$

g

note:

⇒ spread in $V_\parallel$ due drag (h) + diffusion (g)
↓ slow ↓ dispersion

⇒ spread in $V_\perp$ due diffusion (g), only.

Now, can define deflection time by:

$$\tau_D = V^2/(dV_\perp^2/dt)$$

can similarly consider:

→ τ_D for $v \gg v_{Te}, v_{Ti}$ (supra-thermal)

→ $v \sim v_{Ti} < v_{Te}$

→ $v \sim v_{Te} > v_{Ti}$

for supra-thermal:

$$\boxed{\tau_D \sim m_T v^3 / 16\pi n_0 e^2 Z_T^2 \ln\Lambda}$$

others: HW.

Ⓒ Conductivity / Resistivity due Collisions (Spitzer)

Now, for conductivity:

$$\frac{\partial f_e}{\partial t} - \frac{|e|}{m} E \frac{\partial f_e}{\partial v} = \left.\frac{\partial f_e}{\partial t}\right)_c$$

collision operator.

i.e. current to applied field.

stationarity ⇒

$$\frac{|e|}{m} E \int d^3v\, f_e = \int d^3v\, v \left.\frac{\partial f_e}{\partial t}\right)_c$$

→ slowing down problem ⇒ dynamical friction.

electron acceleration balanced by collisional drag (on electrons)

basic physics of resistivity

Quickie:

$$\frac{\partial f}{\partial t} + \frac{q}{m} E \frac{\partial f}{\partial v} = -\nu_{ei}(f - f_0)$$

$$+\frac{|e|}{m_e} E \frac{\partial f_0}{\partial v} = +\nu_{ei}\, \delta f$$

$$\underline{J} = -n_0|e| \int d^3v\, \underline{v}\, \delta f$$

$$= -n_0|e| \int d^3v\, \underline{v} \left(\frac{|e|}{m_e} \underline{E} \cdot \frac{\partial f_0}{\partial \underline{v}} \Big/ \nu_{ei} \right)$$

$$\nu_{ei} \cong \pi n (e^2)^2 \ln\Lambda / m_e^2 v_{th_e}^3$$

$$\boxed{1/\nu_{ei} \sim \frac{m_e^2 v_{th_e}^3}{\pi n (e^2)^2 \ln\Lambda} \sim \frac{T_e^{3/2} \sqrt{m_e}}{\pi n_0 (e^2)^2 \ln\Lambda}}$$

$$\underline{J} \sim \frac{\cancel{n_0}|e|^2}{\cancel{m_e}} E \frac{\cancel{m_e^2} v_{th}^3}{\pi \cancel{n} (e^2)^{\cancel{2}} \ln\Lambda} \sim \frac{T^{3/2}}{\pi \sqrt{m_e}\, e^2 \ln\Lambda}$$

better to:

→ go to electron frame

→ consider ion motion

i.e

$$q_i E = -\frac{\partial}{\partial t}(m_i u_i)_c$$

take $f_i = \delta(\underline{v} - \underline{V})$

{ Cold ion approx → use beam calcn

then

$$q_i E = -\frac{4\pi n e^2 z_i^2}{m_i} \ln\Lambda \frac{\partial}{\partial V}\left[\frac{1}{V}\phi(V/v_{Te})\left(1+\frac{m_i}{m_e}\right)\right]$$

$u < v_{The}$ limit

dynamical friction

→ point is that:

– ions are test
– no ion–ion scattering $(V > v_{Ti})$
just i–e scattering

so, for $V < V_{Te}$, can expand:

$$E = \frac{4\pi n_e e_i^2 q_i}{m_i}\left[\frac{1}{V_{Te}^3}\frac{4}{3\sqrt{\pi}}V\right]\frac{m_i}{m_e}\ln\Lambda$$

$$= \frac{n z_i V}{\sigma} = \frac{J}{\sigma} \quad \text{(frame)}$$

$$\boxed{\sigma = \frac{3 m_e}{16\sqrt{\pi}\, Z e^2 \ln\Lambda} V_{Te}^3}$$

$$\boxed{\sigma \sim \frac{T_e^{3/2}}{Z e^2 \ln\Lambda\, m_e^{1/2}}}$$

~ increases with T

~ indep n

~ charge carriers ↑

but scattering ↑

$$\eta \sim 1/\sigma$$

$$\sim 1/T_e^{3/2}$$

$$\eta = \frac{c^2}{\omega_{pe}^2}\nu_{ei} \sim \frac{\Lambda^0}{T^{3/2}}$$

diffu

⇒ basic collisions/conductivity/resistivity

| good!